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An Application of Mathematical Modeling to Mass Transfer Rate Distribution in a Simple Geometry Using Wind Driven Effect

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Abstract: **A simple mathematical model together with the analysis of mass transfer rate in a simple geometry has been considered and simple analytic techniques was employed and came up with the model of mass transfer rate in a simple geometry using wind driven effect only. Assuming the air temperature within the building to be uniform, this has led to the simplification and calculation of air volumetric rate and mass transfer rate. Various parameters have been used to see the effects of changes of the parameters to the overall flow distribution in the building.**

Keywords: **Mass Transfer Rate, Natural Ventilation, Mechanical Ventilation, Simple Geometry, Air Volumetric Rate, Bernoulli Equation.**

1. INTRODUCTION

In fluid dynamics, the Bernoulli's principle states that for an in-viscid flow, an increase in the speed of the fluid occurs simultaneously with the decrease in pressure or a decrease in the fluid's potential energy. Bernoulli's principles is named after the Dutch-Swiss mathematician Daniel Bernoulli's who published his principle in his book Hydrodynamics in 1738 and was latter derived by Leonhard Euler in 1755. Together Bernoulli and Euler tried to discover more about the flow of fluid. In particular, they wanted to know about the relationship between the speed of which blood flows and its pressure. To investigate this, Daniel experimented by puncturing the wall of a pipe with a small open ended straw and noted that the height to which the fluids rose up the straw was related to fluid's pressure in the pipe (Asmar, 2005).

Soon physician all over Europe were measuring patient blood pressure by sticking point-ended glass tubes directly in to their arteries. It was not until 170 years later, in 1896 that an Italian Doctor discovered a less painful method which is still in used today. However, Bernoulli's method of measuring pressure is still used today in modern aircraft to measure the speed of air passing the plane; that is its speed. The lift forces can be calculated using Bernoulli equations established by Bernoulli over a century before the first man-made wings were used for the purpose. Taking his discoveries further, Daniel Bernoulli now returned his earlier work on conservation of energy. It was knows that a moving body exchanges its kinetic energy for potential energy when it gains height. Daniel realized that in a similar way, a moving fluid exchanges its kinetic energy for pressure. Mathematically this law is now written as:

$$\frac{1}{2}\rho q^2 + p = Constant.$$

Where p is the pressure, rho = ρ is the density of the fluid and q is its velocity. A consequence of the fluid falls. This is exploited by the wing of an aero plane which is designed to create an area of fast flowing air above its surface. The pressure of this area is lower and so the wing is sucked upwards (Polkin et al, 2006).

The mass transfer rate in a simple geometry (single-sided ventilation building) is a special case of multiple connected spaces building through large opening. The possibility of multiple connected spaces leads to a new class of problem, each its own time scale associated with volume and the flow rate through the space and detailed flow patterns depends on the connection between the spaces, while in the simple geometry (single-sided ventilated building) it is one of the more forms of natural ventilation and occur when there is single opening in to a space. It may take the form of either mixing or displacement ventilation depending on the position of the opening. This leads to the mathematical interest in the fluid motion of the fluid particles in naturally ventilated building. In this project we will only limit our study on a simple geometry (i.e., single-sided ventilated building) (Lawan, 2008).

Flow of mass transfer rate between a various zones of a building plays a very important role in the overall thermal balance in buildings, while regulating to a certain degree, it increases the levels of thermal comfort and of indoor air quality. The exchange of mass can achieved either by mechanical means (mechanical ventilation) or through a large opening of the building envelope (natural ventilation).

Calculations of mass transfer rate through large opening or natural ventilated building is a conflicted task. The random nature of the wind makes estimate of mass transfer rate characteristics much more conflicted than in the case of mechanical ventilation. So fluid dynamics provide a meaningful and stable scientific background to calculate the air flow distribution in a simple geometry (i.e. single-sided ventilation building). This can be achieved either by using macroscopic models or microscopic models (Yang, 2004).

2. ASSUMPTIONS AND NOTATIONS

Assumptions

The model assumptions are as follows:

1. The flow is two-layer exchange flow and it cause by wind driven effect.
2. The pressure variation is a function of the distance from the floor level.
3. Uniform interior air temperature.
4. The neutral level (height where the $V_{1,2}$=0) is at the middle of the opening.
5. Steady, in-viscid and incompressible flow.
6. Absolute pressure P_0 at the height (y) of the opening centerline is everywhere equal.
7. Practically the height of the opening (y), ranges from 0 to 2.0m i.e. $0 \leq x \leq 2.0$, width of the opening (w) is 1.0m. experientially the actual value of $C_d = 0.6$, temperature in the zone I and zone 2 assume to be $T_1 = 273k \; and \; T_2 = 283k, with \; \bar{T} = 278k.$
8. At the floor level the height of the opening is zero i.e. $y = 0$ implies $Q(y) = 0$ which is the air flow.

Notations

$\bar{\rho}$ = the mean average density between the inlet and outlet, (kg/m^3),

ρ_1 = density of the outdoor (kg/m^3),

ρ_2 = density of the indoor (kg/m^3),

$\Delta\rho$ = density difference between the indoor and the outdoor $(\mathrm{kg/m^3})$,

g = gravitational acceleration, $10\mathrm{m/s^2}$,

Y = elevation (neutral height),

W= width of the opening,

C_d= discharge coefficient of the opening,

$\frac{dQ}{dY}$ = Mass transfer rate.

3. MASS TRANSFER RATE MODEL

The geometry under consideration is given in figure 1, the geometry has two opposite vertical openings of height "Y" and width equal to "w". The 2-zones of the geometry are zone-1 (out-door) and zone-2 (in-door). The diagram of typical zone geometry is given by,

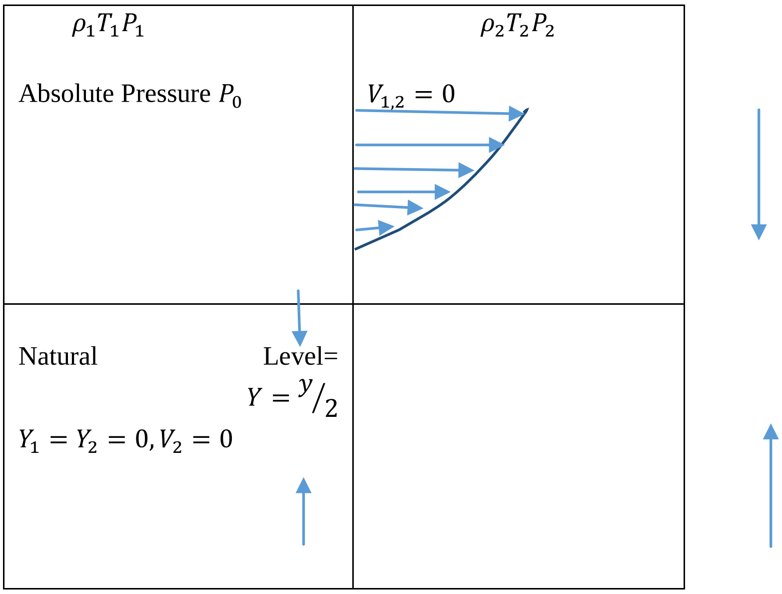

Figure 1:

Consider the simple geometry in figure 1.

At zone-1, the pressure P_1 at the level height Y is;

$$P_1 = P_0 + \rho_1 g Y \tag{1}$$

At zone-2, the pressure P_2at the level height Y is;

$$P_2 = P_0 + \rho_2 g Y \tag{2}$$

$$\therefore P_2 - P_1 = (\rho_2 - \rho_1) g Y = \Delta P = \Delta \rho g Y \tag{3}$$

The Bernoulli equation at zone-1 is;

$$P_1 + \rho_1 g Y_1 + \frac{1}{2} \rho_1 V_1^2 = constant \tag{4}$$

While at zone-2 is;

$$P_2 + \rho_2 g Y_2 + \frac{1}{2} \rho_2 V_2^2 = constant \tag{5}$$

Comparing equation (4) and (5) we have;

$$P_1 + \rho_1 g Y_1 + \frac{1}{2} \rho_{1V_1^2} = P_2 + \rho_2 g Y_2 + \frac{1}{2} \rho_{2V_2^2} \tag{6}$$

Since $\quad Y_1 = Y_2 = 0 \text{ and } V_2 = 0$

$$\therefore P_1 + 0 + \frac{1}{2} \rho_1 V_1^2 = P_2 + 0 + \frac{1}{2} \rho_2 V_2^2 \tag{7}$$

$$\therefore P_2 - P_1 = \frac{1}{2} \rho_1 V_1^2 \tag{8}$$

Let $\quad V_1 = V$

$$\therefore 2\Delta\rho gY = \rho_1 V^2$$

Implies $V^2 = \frac{2\Delta\rho gY}{\rho_1}$ (9)

So, the average mean velocity of air in the two zones is,

$V_{1,2}^2 = \frac{2\Delta\rho gY}{\rho_1}$ (10)

Let $\rho_1 = \bar{\rho} = \frac{\rho_2+\rho_1}{2}$

This is the mean average density.

Now,

$V_{1,2} = \sqrt{\frac{2\Delta\rho gY}{\bar{\rho}}}$ (11)

By P.F. linden et.al (1995), the reduced gravity is given by;

$$g\frac{\Delta\rho}{\bar{\rho}} = g\frac{T}{\bar{T}}$$

This is the fractional density difference due to the temperature differences.

$\therefore V_{1,2} = \sqrt{\frac{2\Delta TgY}{\bar{T}}}$ (12)

The flow is proportional to the elemental area of the openings and the mean average velocity of the air.

Mass transfer rate $= Elemental\ area \times V_{1,2}$

$dQ \propto wdY\sqrt{\frac{2\Delta TgY}{\bar{T}}}$ (13)

If C_d is inserted in (3.2.13) we have;

$\frac{dQ}{dY} = wC_d\sqrt{\frac{2\Delta TgY}{\bar{T}}}$ (14)

This is the required model equation of mass transfer rate.

4. SOLUTION OF THE MODEL

Let U be the dummy variable.

$dQ = wC_d\sqrt{\frac{2\Delta\rho gU}{\bar{\rho}}}dU$ (15)

Since the equation is first order linear differential equation, we solve using separation of variable. Integrating through, we have;

$Q(y) = wC_d\int_{u=0}^{u=\frac{y}{2}}\sqrt{\frac{2\Delta\rho gU}{\bar{\rho}}}dU$ (16)

$Q(y) = wC_d\sqrt{\frac{2\Delta\rho g}{\bar{\rho}}}\int_{u=0}^{u=\frac{y}{2}}U^{\frac{1}{2}}dU$ (17)

$Q(y) = wC_d\sqrt{\frac{2\Delta\rho g}{\bar{\rho}}}.\frac{2}{3}\left[U^{\frac{3}{2}}\right] + C$ (18)

$Q(y) = \frac{2}{3}wC_d\sqrt{\frac{2\Delta\rho g}{\bar{\rho}}}.\left[\frac{Y}{2}\right] + C$ (19)

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$$Q(y) = \frac{1}{3} w C_d \sqrt{\frac{\Delta\rho g}{\bar{\rho}}} . y^{1.5} + C \quad (20)$$

Initial condition;

Q(y) = 0, at y = 0; then $C = 0$.

$$\therefore Q(y) = \frac{1}{3} w C_d \sqrt{\frac{\Delta\rho}{\bar{\rho}}} . y^{1.5} \quad (21)$$

But,

$$\frac{dm}{dy} = Q(y)\bar{\rho} \quad (22)$$

Implies,

$$\frac{dm}{dy} = \frac{1}{3} C_d \bar{\rho} w \sqrt{\frac{g\Delta\rho}{\bar{\rho}}} \cdot y^{1.5} \quad (23)$$

$$dm = \frac{1}{3} C_d \bar{\rho} w \sqrt{\frac{g\Delta\rho}{\bar{\rho}}} \cdot y^{1.5} dy \quad (24)$$

By taking the integral of both sides we have,

$$\int dm = \int \frac{1}{3} C_d \bar{\rho} w \sqrt{\frac{g\Delta\rho}{\bar{\rho}}} \cdot y^{1.5} dy \quad (25)$$

$$m(y) = \frac{1}{3} C_d \bar{\rho} w \sqrt{\frac{g\Delta\rho}{\bar{\rho}}} \cdot \int y^{1.5}\, dy + c \quad (26)$$

$$m(y) = \frac{1}{3} C_d \bar{\rho} w \sqrt{\frac{g\Delta\rho}{\bar{\rho}}} [y] \cdot \frac{2}{5} + c \quad (27)$$

$$m(y) = \frac{1}{3} C_d \bar{\rho} w \sqrt{\frac{g\Delta\rho}{\bar{\rho}}} \cdot [\frac{y^{\frac{5}{2}}}{2}] \cdot \frac{2}{5} + c \quad (28)$$

$$m(y) = \frac{2}{15} C_d \bar{\rho} w \sqrt{\frac{g\Delta\rho}{\bar{\rho}}} \cdot y^{\frac{5}{2}} + c \quad (29)$$

$$m(y) = \frac{2}{15} C_d \bar{\rho} w \sqrt{\frac{g\Delta\rho}{\bar{\rho}}} \cdot y^{2.5} + c \quad (30)$$

Initial conditions,

$m(y) = 0$, at y = 0.

Therefore,

$$m(y) = \frac{2}{15} C_d \bar{\rho} w \sqrt{\frac{g\Delta\rho}{\bar{\rho}}} \cdot y^{2.5} \quad (31)$$

$$\therefore m(y) = \frac{2}{15} C_d \bar{\rho} w \sqrt{\frac{g\Delta\rho}{\bar{\rho}}} \cdot y^{2.5} \quad (32)$$

This is the required solution of the model.

5. CONCLUSION

Flow of mass transfer rate between a various zones of a building plays a very important role in the overall thermal balance in buildings, while regulating to a certain degree, it increases the levels of thermal comfort and of indoor air quality. The exchange of mass can achieved either by mechanical means (mechanical ventilation) or through a large opening of the building envelope (natural ventilation).

From the discussion we saw that if the values of discharge coefficients decreases then, the mass transfer rate will also decreases, while in the case of higher value of discharge coefficient the mass transfer rate wills increases.

Also if the temperature increases, the mass transfer rate will decreases while decreases in temperature will increases the mass transfer rate within the geometry.

Therefore, to have best and effective ventilation in the building the width of the opening has to be increases and the temperature within the building will be reducing.

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